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Electron excitation by a multi passage laser beam in a plasma

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Abstract. The limits put by optical guiding, and channel guiding mechanisms on the Laser Wakefield Acceleration (LWFA) technique are imposed on the Resonant Laser Wakefield Acceleration (RLWFA) scheme. Energy gained by the electrons in both schemes are calculated and compared. It has presented that in the RLWFA case, the electrons gain more and more energy after each traversal of the laser pulse and the electrons in a plasma gain about 3 GeV after 10 passages of the laser pulse. They gain 13 GeV when the laser light makes 50 passages and 26 GeV after the laser beam traverses the plasma 100 times. Moreover, the channel guiding mechanism is integrated to the RLWFA scheme and together with diffraction guiding a model for electron acceleration is proposed.

PACS. 52.40.Mj Particle beam interactions in plasma – 52.40.Nk Laser-plasma interactions (e.g., anomalous absorption, backscattering, magnetic field generation, fast particle generation)

1 Introduction

Accelerating the electrons in a plasma by using electromagnetic fields has been an interesting problem since 1979, when Tajima and Dawson [1] proposed the mechanism. There are three promising mechanisms of particle acceleration in plasmas: the Plasma Wakefield Acceleration (PWFA), the Beat Wave Acceleration (BWA) and the Laser Wakefield Acceleration (LWFA). In all methods, the main purpose is to generate a wakefield by exciting a longitudinal wave in the plasma. The PWFA method, unlike the BWA and LWFA mechanisms, requires an electron beam to drive the plasma wave. However, in the rest of the mechanisms, excitation is performed by laser beams.

In the case of BWA, two laser beams with different frequencies, ω_1 and ω_2 are used to create a Langmuir wave in the plasma. Therefore, the achievement of the resonance condition, $\Delta\omega = \omega_\mathrm{p}$, ($\Delta\omega = \omega_1 - \omega_2$, and ω_p is the plasma frequency) is necessary. LWFA method, on the other hand, is a unique one to generate a wakefield in the plasma without the requirement of the resonance condition. A single, short and high power laser beam is enough to create a wake in the plasma.

The resonant excitation of a plasma wave is first proposed and performed for the PWFA case [2]. Instead of multiple electron bunches, the same mechanism is adopted for multiple laser pulses and a resonant acceleration of electrons in a plasma is demonstrated both analytically and numerically [3,4], using quasistatic approximation in a one dimensional consideration. Furthermore, in some studies [5,6], an optimized train of independently adjustable laser pulses are used to generate large amplitude plasma waves. The resonant PWFA mechanism needs a long collinear sequence of multiple electron bunches whose period has to be the multiples of the plasma wavelength. The resonant excitation of the plasma waves can also be achieved by a single laser pulse as in the case of LWFA. In analogy with the resonant PWFA case, RLWFA method, which is considered in this work, can be presented by using a single laser pulse, instead of multiple electron bunches, making round-trip propagation in the plasma located in an optical resonator.

The physical mechanism for the wakefield generation in the RLWFA is that, after the first passage of the laser beam, there is a wakefield generated behind the laser pulse. When the laser beam passed over the plasma and completed its cycle in the resonator, it encounters the previously excited wakefield and enhances it. Therefore, the wakefield generated behind the each traversal of the laser pulse grows to a larger amplitude when the plasma waves are superposed.

There are some guiding mechanisms, proposed to improve the energy gain of the electrons in the LWFA method. Those guiding mechanisms like diffraction guiding, optical guiding, etc. are all well established for the LWFA case. The mechanisms of the arrangement of either the laser beam or the plasma parameters in order to increase the energy gain, are all applicable for the RLWFA case as well. However, the physical situation in the plasma after each traversal of the laser beam must be well understood.

Assuming cold, unmagnetized plasma with an electron fluid density n_e and stationary ions, the momentum, continuity and Poisson's equations yield a forced, damped

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$$E_z\left(z,t+\frac{ML}{v_\mathrm{p}}\right)=E_{z0}\sum_{m=0}^{M/2}\exp\left[\alpha\left(z-v_\mathrm{p}\left(t+\frac{ML}{v_\mathrm{p}}\right)\right)\right]\cos\left[k\left[z-v_\mathrm{p}\left(t+\frac{ML}{v_\mathrm{p}}\right)\right]+\psi_0\right]$$
$$+\sum_{m=0}^{M/2}\exp\left[-\alpha\left(z+v_\mathrm{p}\left(t+\frac{ML}{v_\mathrm{p}}\right)\right)\right]\cos\left[k\left[z+v_\mathrm{p}\left(t+\frac{ML}{v_\mathrm{p}}\right)\right]+\psi_0\right]\quad(6)$$

harmonic oscillator equation for the wake potential

$$\frac{\partial^2\phi}{\partial t^2}+\upsilon_\mathrm{ei}\frac{\partial\phi}{\partial t}+\phi\omega_\mathrm{p}^2=-\frac{\omega_\mathrm{p}^2}{e}\phi_\mathrm{NL}\quad(1)$$

where υ_ei is the electron ion collision frequency and ϕ_NL is the non-linear potential describing the ponderomotive force. Inserting the ponderomotive potential [7] $\phi_\mathrm{NL}=-(mc^2/2)\left|\mathbf{a}^2(r,z,t)\right|$, and assuming a bi-Gaussian laser beam, with $\mathbf{a}$ being the normalized vector potential, the wakefields in the axial and radial directions are found as follows [8]

$$E_z(r,z,t)=\frac{\sqrt{\pi}}{4e}mc^2a_0^2\sigma_zk_\mathrm{p}^2\exp\left(-\frac{k^2\sigma_z^2}{4}-\frac{r^2}{\sigma_r^2}\right)\times\cos k(z-v_\mathrm{p}t)\quad(2)$$

$$E_r(r,z,t)=-\sqrt{\pi}\frac{mc^2}{e}a_0^2\frac{r}{\sigma_r^2}\sigma_z\exp\left(-\frac{2r^2}{\sigma_r^2}+\frac{k_p^2\sigma_z^2}{4}\right)\times\sin k(z-v_\mathrm{p}t)\quad(3)$$

where, σ_z and σ_r are the rms values of the axial pulse length and radial spot size of the beam, respectively.

While the radial wakefield pushes the electrons inside the channel in which they are accelerated, the axial one is responsible for the acceleration phenomena. In the resonant excitation case the laser beam propagates in a plasma in either open or closed ring configuration as shown in Figure 1 [9]. Therefore, the wakefield grows continuously in each traversal as shown in Figure 2. A single pass wave field behind a laser pulse propagating to the positive and negative directions can be expressed in terms of equation (2)

$$E_z(z,t)=E_{z0}\exp\left[\pm\alpha(z-v_\mathrm{p}t)\right]\cos\left[k(z\pm v_\mathrm{p}t)+\psi_0\right]\quad(4)$$

where E_{z0} is the amplitude on axis and ψ_0 is an appropriate phase constant. Assuming that the laser beam makes M round trip propagation between two mirrors with a spacing of L, the wakefield is then expressed in terms of a series expression:

$$E_z\left(z,t+\frac{ML}{v_\mathrm{p}}\right)=E_{z0}\sum_{m=0}^{M}\exp\left[\pm\alpha\left(z-v_\mathrm{p}\left(t+\frac{ML}{v_\mathrm{p}}\right)\right)\right]\times\cos\left[k\left[z\pm v_\mathrm{p}\left(t+\frac{ML}{v_\mathrm{p}}\right)\right]+\psi_0\right]\quad(5)$$

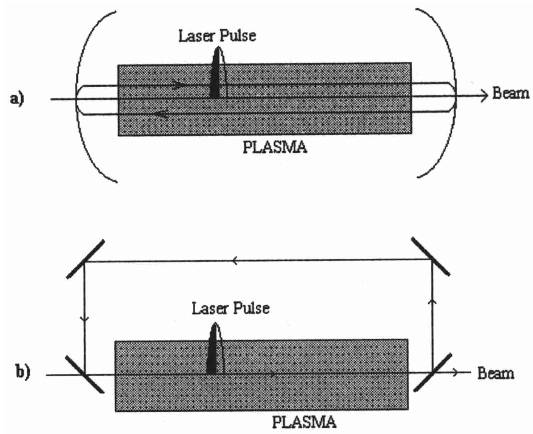

Fig. 1. The resonant excitation of plasma waves can be achieved in a configuration similar to the optical resonator. It can be either open resonator configuration (a), or closed one (b).

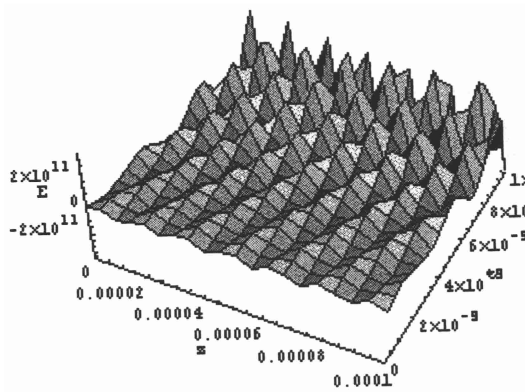

Fig. 2. The longitudinal wakefield in V/m after the laser light has passed plasma 100 times.

since the pulse is propagating in the positive and negative directions equation (5) must be expressed in such a way that it should exhibit such a behavior:

see equation (6) above.

2 Diffraction guiding

When the laser beam encounters the plasma media, due to a remarkable index of refraction, it is diffracted. Hence, diffraction puts a limit on the acceleration mechanism. This limit is explained by the concept of Rayleigh length that is the distance beyond which the radiation field is

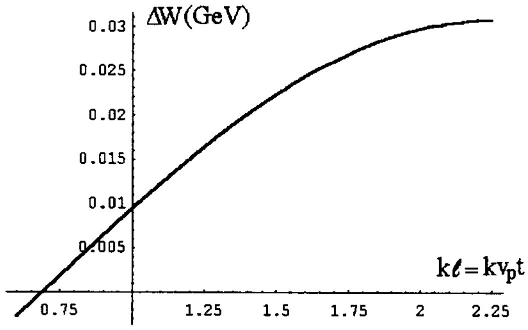

Fig. 3. The energy gained by electrons in the single pass case in GeV with respect to dimensionless parameter that is the product of the wave number, phase velocity and time.

a good approximation to be the total field. Therefore, Rayleigh length characterizes the distance over which the laser beam spreads transversely [10]. Since there is a limit due to diffraction on the interaction distance, the maximum energy gain should be calculated within the Rayleigh length

$$\Delta W = eE_{z0}\int_0^{\pi Z_{\rm R}} \exp\left[\pm\alpha(z-v_{\rm p}t)\right] \times \cos\left[k(z-v_{\rm p}t)+\psi_0\right]{\rm d}z \quad (7)$$

where $Z_{\rm R} = \pi R_0^2/\lambda_0$ is the Rayleigh length, and $\psi_0 = \tan^{-1}(\alpha/k)$ is the phase constant. The integral in equation (7) can easily be evaluated and the result can be presented in the form of trigonometric functions, as follows:

$$\Delta W = \frac{eE_{z0}}{k}\left[\sin(k\pi Z_{\rm R}+v_{\rm p}t+\psi_0)-\sin(v_{\rm p}t+\psi_0)\right]. \quad (8)$$

The energy gain expressed in equation (8) strongly depends on the interaction length which is the product of the phase velocity and time and also the phase constant. The collisionless plasma assumption implies that $\alpha = 0$, and the phase constant vanishes automatically. Hence the energy gain for a single pass wave is expressed in terms of the propagation length, $\ell = v_{\rm p}t$ and the Rayleigh length

$$\Delta W = \frac{eE_{z0}}{k}\left[\sin(k\ell)-\sin(k\pi Z_{\rm R}+k\ell)\right] \quad (9)$$

for some numerical values like, $E_{z0} = 1.9\times10^{10}$ V/m and $k\pi Z_{\rm R} = 113$ the energy gain as presented in Figure 3, is found to be ~ 0.02 GeV in maximum within the Rayleigh length.

In order to see the significance of the resonant case, the same calculation is carried out for the energy gain. In this case the gain along the Rayleigh length is expressed in a series expression since the wakefields are expressed in the same manner

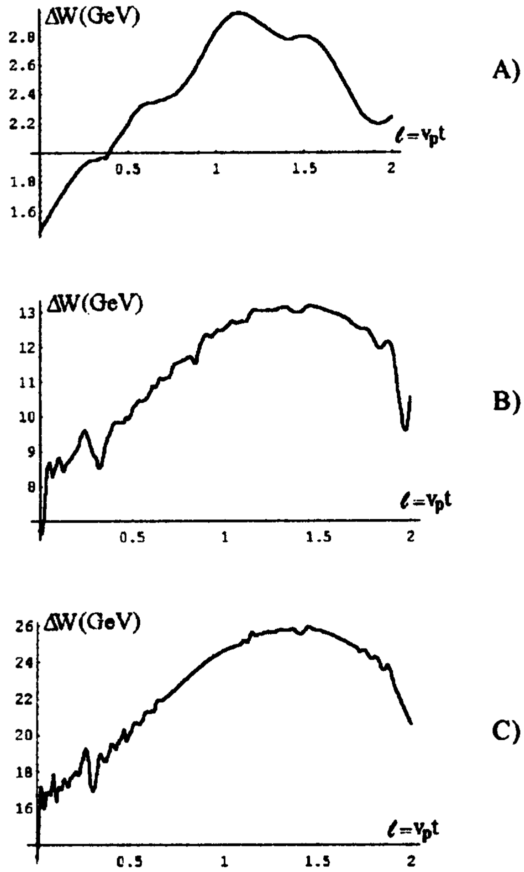

Fig. 4. The energy gained by electrons in GeV with respect to interaction length after the laser pulse traversed plasma, (A) 10 times, (B) 50 times and (C) 100 times.

$$\Delta W = \sum_{m=0}^{M/2}\exp({\rm i}kmL) \times \int_0^{\pi Z_R}\Big\{\cosh\left[\alpha(z-v_{\rm p}t)\right]\cos\left[k(z-v_{\rm p}t)+\psi_0\right] + \exp(-{\rm i}kz+\alpha v_{\rm p}t)\cos(kv_{\rm p}t-\psi_0)\Big\}{\rm d}z. \quad (10)$$

Carrying on the same procedure as it was done for the previous case, the energy gain is estimated in a series expression of trigonometric functions

$$\Delta W = \sum_{m=0}^{M/2}\frac{eE_{z0}}{k}\Big\{\left[\cos(k\ell)\sin\left[k(\pi Z_{\rm R}+mL)\right]\right] + \cos(kmL)\sin\left[k(\pi Z_{\rm R}-\ell)\right]+\sin\left[k(mL+\ell)\right]\Big\}. \quad (11)$$

It is shown in Figure 4 that, when a laser beam propagates in a plasma 10 times, the energy gain is 2.8 GeV, when the beam passes 50 times, the energy gained by electrons becomes 13 GeV and after 100 times electrons gain 26 GeV of energy.

3 Optical guiding

Since the most significant optical parameter of plasma is the refractive index, the optical guiding is based on the principle of refractive guiding. The idea is arranging the index of refraction in such a way that to increase the energy gain. There is a well-known criterion for refractive guiding that the radial profile of index of refraction must exhibit a maximum on axis [11]. That is $\partial\eta(r)/\partial r < 0$. This implies that the phase velocity along the propagation axis is less than it is off-axis value [12]. In general, the index of refraction is defined in terms of the plasma frequency, laser frequency and the ambient density

$$\eta(r) = 1 - \frac{\omega_{\mathrm{p}}^2}{2\omega_0^2}\frac{n(r)}{n_0}\frac{1}{1+\phi_{\mathrm{s}}} \tag{12}$$

where ϕ_{s} is the slow part of the potential, n_0 is the ambient density, $n(r)$ is the radial density profile, and ω_{p} and ω_0 are the plasma and laser frequencies, respectively. The slow part of the potential is expressed in terms of the packet shape or the normalized vector potential, $\mathbf{a}$. Therefore, the index of refraction is explicitly written as:

$$\eta(r) = 1 - \frac{\omega_{\mathrm{p}}^2}{2\omega_0^2}\frac{n(r)}{n_0}\left(1+\frac{|\mathbf{a}|^2}{2}\right)^{-1/2}. \tag{13}$$

After differentiating equation (13), the above-mentioned condition on the index of refraction turns out to be a general correlation between the plasma density profile and the laser beam packet shape

$$\frac{2}{n(r)}\frac{\partial n(r)}{\partial r} < |\mathbf{a}|\frac{\partial|\mathbf{a}|}{\partial r}. \tag{14}$$

Integration of equation (14) gives a detailed condition for refractive guiding. For the bi-Gaussian beam case, the condition can be derived explicitly, as follows.

The condition for the optical guiding implies that the regions, at which the acceleration takes place, strongly depends on the radial profile of the refractive index. Neglecting ξ, in comparison with σ_z, and taking the derivative of η with respect to r, it is found that,

$$\frac{\partial\eta(r)}{\partial r} = \frac{\omega_0^2}{2\omega_{\mathrm{p}}^2}\frac{1}{n_0}\left\{-\frac{\partial n(r)}{\partial r}\left[1+\frac{1}{2}a_0^2\exp\left(-\frac{r^2}{\sigma_r^2}\right)\right]^{-1/2} - \frac{1}{2}n(r)\left(\frac{2r}{\sigma_r^2}\right)a_0^2\exp\left(-\frac{r^2}{\sigma_r^2}\right) \times\left[1+\frac{1}{2}a_0^2\exp\left(-\frac{r^2}{\sigma_r^2}\right)\right]^{-3/2}\right\}. \tag{15}$$

Hence introducing the decreasing condition for the index of refraction, a condition for the density profile is found to be,

$$-\int\frac{\partial n}{n} < \frac{1}{2}\int\frac{\left[a_0^2\frac{2r}{\sigma_r^2}\exp\left(-\frac{2r^2}{\sigma_r^2}\right)\right]}{\left[1+\frac{1}{2}a_0^2\exp\left(-\frac{2r^2}{\sigma_r^2}\right)\right]}\mathrm{d}r. \tag{16}$$

Integration yields,

$$-\ln|n(r)| < -\ln\left|1+\frac{1}{2}a_0^2\exp\left(-\frac{r^2}{\sigma_r^2}\right)\right|^{1/2} \tag{17}$$

which gives a condition for the electron density in terms of the laser parameters

$$n(r) < \sqrt{1+\frac{1}{2}a_0^2\exp\left(-\frac{r^2}{\sigma_r^2}\right)}. \tag{18}$$

Since the refractive guiding does not depend on the wakefields there are no differences between the LWFA and RWFA cases. Thus, as in the LWFA case equation (18) describes the upper limit for a density profile. Therefore, refractive guiding in either LWFA or RWFA automatically satisfies the channel guiding.

4 Discussion and conclusion

It is known that for accelerating the electrons by a laser beam, before the laser beam losses half of its original energy; the electrons must stay in phase with the plasma oscillations within the Rayleigh length. In order to understand the validity of limitations [8,10] in the case of Resonant Laser Wakefield Acceleration (RLWFA) mechanism, the guiding mechanisms have been taken into consideration again. The treatment of these limits shows that they are not affected by the multiple passages of the laser beam. The Rayleigh length and phase detuning distances are calculated numerically for the present application. It has been seen that for a plasma wavelength of $\lambda_{\mathrm{p}} = 68$ μm and laser wavelength of $\lambda_0 = 780$ nm with a spot size at the focus $R_0 = 10$ μm, the RL is $\pi Z_{\mathrm{R}} = \pi^2 R_0^2/\lambda_0 = 1.26$ mm. Whereas, the phase detuning distance (L_ϕ), $L_\phi = 5.1$ cm. Since RL is less than the phase detuning distance, it is kept to be the upper limit for the interaction distance in all energy calculations.

The LWFA scheme exceeds MeV of energy limit even in a single pass wave field. As it is demonstrated, electrons gain about 0.03 GeV of energy in a Rayleigh length, when the laser beam traverses plasma once. The energy gain profile, which has a sinusoidal behavior, is plotted with respect to a dimensionless parameter ($k\ell$) and it is seen that after a certain value of ($k\ell$) the electrons start gaining energy. That value was a point of interest and it shows that the electrons gain energy for the points satisfying,

$$k\ell > \tan^{-1}\left(\frac{\sin k\pi Z_{\mathrm{R}}}{1-\cos k\pi Z_{\mathrm{R}}}\right).$$

Since the Rayleigh length and the wave number of the laser pulse are all known quantities, this inequality allows us to compute the interaction length along which electrons gain more energy. The interaction length, which is the product of the phase velocity and time, is about a micrometer. On the other hand, the rate at which electrons gain energy, when the laser beam encounters plasma once is 0.03 GeV/μm, or it is about 3 TeV/m.

When RLWFA is taken into consideration, although the laser beam is allowed to travel in plasma many times, it is obvious that Rayleigh length limits the interaction length due to the diffraction phenomena in the plasma. However, in this case diffraction guiding is not enough for electrons to gain more and more energy. The philosophy behind RLWFA is that if the laser pulse encounters a larger amplitude plasma oscillation in the plasma after each trip in the resonator, it will enlarge those oscillations furthermore. It has known that those oscillations continue after the laser beam has passed over. However, plasma oscillation is a necessary but not a sufficient condition in order to accelerate the electrons. The electron oscillations must be continuous and in phase with the laser beam when it interacts plasma once more. Therefore, the optical guiding must be satisfied as well as the diffraction guiding so that electrons gain more energy after each time that the laser pulse encounters the plasma. In other words, RLWFA is useful if plasma has an appropriate density distribution on the axis.

After all those requirements satisfied, the energy gain is calculated in the same way as it was done for LWFA case, but this time, the expression of interest is in the form of a series expression and contains the effects of laser light propagating both in the positive direction and in opposite direction. Thus calculation for the energy gained by electrons which are directed parallel to the laser beam, like in a closed resonator configuration is demonstrated in Figure 2, which shows that the electrons gain about 3 GeV of energy after 10 passages of the laser beam. The energy gain increases up to 13 GeV when the beam traverses the plasma 50 times and it reaches a large value like 26 GeV when the beam travels in the plasma 100 times. As it is shown in Figure 4, for all cases although the entire distribution of energy gain increases, there exists some minor fluctuations. Those fluctuations show that the energy gain is not a smooth process. An intense consideration of equation (11) presents that the monotony of the energy gain in RLWFA scheme requires a correlation between the interaction length, length of the plasma and the parameter m, which is describing how many times that the laser pulse traversed the plasma. It can be demonstrated analytically as:

$$\text{(a)}\ \tan(k\ell) < \sum_{m=0}^{M/2} \frac{(1-\cos(k\pi Z_R))}{(1+\cos(k\pi Z_R))} \cot(km\ell),$$

$$\text{(b)}\ \tan(k\ell) = \sum_{m=0}^{M/2} \frac{(1-\cos(k\pi Z_R))}{(1+\cos(k\pi Z_R))} \cot(km\ell),$$

$$\text{(c)}\ \tan(k\ell) > \sum_{m=0}^{M/2} \frac{(1-\cos(k\pi Z_R))}{(1+\cos(k\pi Z_R))} \cot(km\ell).$$

When condition (a) is satisfied, the energy gained by the electrons, increases monotonically and in the case of (c), it decreases. In the critical condition, by the case (b) there exists either a local minimum or a maximum.

Therefore, RLWFA scheme facilitates the production of higher wakefields in plasmas. However, without guiding mechanisms the energy that the electrons gain in the plasma, in other words the acceleration rate, is limited. Making use of the guiding mechanisms can surpass those limits.

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